1 Преобразования числовых выражений

Вычислить:

1. $\dfrac{\sqrt{7{,}2\cdot 2{,}9}\left(\sqrt{\dfrac{7{,}2}{2{,}9}}-\sqrt{\dfrac{2{,}9}{7{,}2}}\right)}{\sqrt{(7{,}2+2{,}9)^2-4\cdot 7{,}2\cdot 2{,}9}}$.

Решение.

$$\frac{\sqrt{7{,}2\cdot 2{,}9}\left(\sqrt{\dfrac{7{,}2}{2{,}9}}-\sqrt{\dfrac{2{,}9}{7{,}2}}\right)}{\sqrt{(7{,}2+2{,}9)^2-4\cdot 7{,}2\cdot 2{,}9}}=\frac{\sqrt{7{,}2\cdot 2{,}9}\,\dfrac{\sqrt{7{,}2^2}-\sqrt{2{,}9^2}}{\sqrt{7{,}2\cdot 2{,}9}}}{\sqrt{7{,}2^2+2\cdot 7{,}2\cdot 2{,}9+2{,}9^2-4\cdot 7{,}2\cdot 2{,}9}}=\frac{7{,}2-2{,}9}{\sqrt{(7{,}2-2{,}9)^2}}=1.$$

Ответ: 1.

2. $\left(\sqrt{\left(\sqrt{5}-\dfrac{5}{2}\right)^2}-\sqrt[3]{\left(\dfrac{3}{2}-\sqrt{5}\right)^3}\right)^{\frac{1}{2}}$.

Решение.

$$\left(\sqrt{\left(\sqrt{5}-\frac{5}{2}\right)^2}-\sqrt[3]{\left(\frac{3}{2}-\sqrt{5}\right)^3}\right)^{\frac{1}{2}}=\left(|\sqrt{5}-2{,}5|-(1{,}5-\sqrt{5})\right)^{\frac{1}{2}}=(2{,}5-\sqrt{5}-1{,}5+\sqrt{5})^{\frac{1}{2}}=1^{\frac{1}{2}}=1.$$

Ответ: 1.

3. $\left(\sqrt{2+\sqrt{3}}+\sqrt{2-\sqrt{3}}\right)^2$.

Решение.

$$\left(\sqrt{2+\sqrt{3}}+\sqrt{2-\sqrt{3}}\right)^2=\left(\sqrt{2+\sqrt{3}}\right)^2+2\sqrt{2+\sqrt{3}}\sqrt{2-\sqrt{3}}+\left(\sqrt{2-\sqrt{3}}\right)^2=2+\sqrt{3}+$$

$$+2\sqrt{(2+\sqrt{3})(2-\sqrt{3})}+2-\sqrt{3}=4+2\sqrt{4-3}=6.$$

Ответ: 6.

4. $\dfrac{2^{-2}+5^0+\left(\dfrac{4}{3}\right)^{-1}}{(0{,}5)^{-2}-5(-2)^{-2}+\left(\dfrac{2}{3}\right)^{-2}-4^1}$.

Решение.

$$\frac{2^{-2}+5^0+\left(\frac{4}{3}\right)^{-1}}{(0{,}5)^{-2}-5(-2)^{-2}+\left(\frac{2}{3}\right)^{-2}-4^1}=\frac{\frac{1}{2^2}+1+\frac{3}{4}}{\frac{1}{(0{,}5)^2}-\frac{5}{(-2)^2}+\left(\frac{3}{2}\right)^2-4}=\frac{\frac{1}{4}+1+\frac{3}{4}}{\frac{1}{0{,}25}-\frac{5}{4}+\frac{9}{4}-4}=\frac{2}{4+1-4}=2.$$

Ответ: 2.

5. $\dfrac{3\sqrt{12}}{\sqrt{45}-4\sqrt{3}}+5\sqrt{2{,}4}(\sqrt{15}+3)$.

Решение.

$$\frac{3\sqrt{12}}{\sqrt{45}-4\sqrt{3}}+5\sqrt{2{,}4}(\sqrt{15}+3)=\frac{3\sqrt{4\cdot 3}}{\sqrt{3\cdot 15}-4\sqrt{3}}+5\sqrt{\frac{12}{5}}(\sqrt{15}+3)=\frac{6\sqrt{3}}{\sqrt{3}\sqrt{15}-4\sqrt{3}}+\frac{5\sqrt{4\cdot 3}}{\sqrt{5}}(\sqrt{15}+3)=$$

$$=\frac{6\sqrt{3}}{\sqrt{3}(\sqrt{15}-4)}+2\sqrt{15}(\sqrt{15}+3)=\frac{6}{\sqrt{15}-4}+30+6\sqrt{15}=\frac{6(\sqrt{15}+4)}{(\sqrt{15}-4)(\sqrt{15}+4)}+30+6\sqrt{15}=$$

$$= \frac{6(\sqrt{15}+4)}{15-16} + 30 + 6\sqrt{15} = -6\sqrt{15} - 24 + 30 + 6\sqrt{15} = 6.$$

Ответ: 6.

6. $3\left(\frac{2}{\sqrt{10}+5} + \frac{5}{\sqrt{10}-2} - \frac{7}{\sqrt{10}}\right)$.

Решение.

$$3\left(\frac{2}{\sqrt{10}+5} + \frac{5}{\sqrt{10}-2} - \frac{7}{\sqrt{10}}\right) = 3\left(\frac{2(5-\sqrt{10})}{(5-\sqrt{10})(5+\sqrt{10})} + \frac{5(\sqrt{10}+2)}{(\sqrt{10}-2)(\sqrt{10}+2)} - \frac{7\sqrt{10}}{10}\right) =$$

$$= 3\left(\frac{2(5-\sqrt{10}}{25-10} + \frac{5(\sqrt{10}+2)}{10-4} - \frac{7\sqrt{10}}{10}\right) = 3\left(\frac{4(5-\sqrt{10})}{30} + \frac{25(\sqrt{10}+2)}{30} - \frac{21\sqrt{10}}{30}\right) =$$

$$= \frac{20-4\sqrt{10}+25\sqrt{10}+50-21\sqrt{10}}{10} = \frac{70}{10} = 7.$$

Ответ: 7.

7. $\left(\frac{15}{\sqrt{6}+1} + \frac{4}{\sqrt{6}-2} - \frac{12}{3-\sqrt{6}}\right)(\sqrt{6}+11)$.

Решение.

$$\left(\frac{15}{\sqrt{6}+1} + \frac{4}{\sqrt{6}-2} - \frac{12}{3-\sqrt{6}}\right)(\sqrt{6}+11) = \left(\frac{15(\sqrt{6}-1)}{(\sqrt{6}+1)(\sqrt{6}-1)} + \frac{4(\sqrt{6}+2)}{(\sqrt{6}-2)(\sqrt{6}+2)} - \frac{12(3+\sqrt{6})}{(3-\sqrt{6})(3+\sqrt{6}}\right) \times$$

$$\times(\sqrt{6}+11) = \left(\frac{15(\sqrt{6}-1)}{5} + \frac{4(\sqrt{6}+2)}{2} - \frac{12(3+\sqrt{6}}{3}\right)(\sqrt{6}+11) = (3\sqrt{6}-3+2\sqrt{6}+4-12-4\sqrt{6})(\sqrt{6}+11) =$$

$$= (\sqrt{6}-11)(\sqrt{6}+11) = -115.$$

Ответ: –115.

8. $\frac{7\sqrt{5-2\sqrt{6}}(5+2\sqrt{6})(49-20\sqrt{6})}{\sqrt{27}-3\sqrt{18}+3\sqrt{12}-\sqrt{8}}$.

Решение.

$$\frac{7\sqrt{5-2\sqrt{6}}(5+2\sqrt{6})(49-20\sqrt{6})}{\sqrt{27}-3\sqrt{18}+3\sqrt{12}-\sqrt{8}} = \frac{7\sqrt{3-2\sqrt{3\cdot 2}+2}(3+2\sqrt{3\cdot 2}+2)(49-20\sqrt{6})}{\sqrt{9\cdot 3}-3\sqrt{9\cdot 2}+3\sqrt{4\cdot 3}-\sqrt{4\cdot 2}} =$$

$$= \frac{7\sqrt{(\sqrt{3}-\sqrt{2})^2}\cdot(\sqrt{3}+\sqrt{2})^2(49-20\sqrt{6})}{3\sqrt{3}-9\sqrt{2}+6\sqrt{3}-2\sqrt{2}} = \frac{7(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})^2(49-20\sqrt{6})}{9\sqrt{3}-11\sqrt{2}} =$$

$$= \frac{7(3-2)(\sqrt{3}+\sqrt{2})(49-20\sqrt{6})}{9\sqrt{3}-11\sqrt{2}} = \frac{7(49\sqrt{3}-20\sqrt{18}+49\sqrt{2}-20\sqrt{12})}{9\sqrt{3}-11\sqrt{2}} = \frac{7(49\sqrt{3}-60\sqrt{2}+49\sqrt{2}-40\sqrt{3})}{9\sqrt{3}-11\sqrt{2}} =$$

$$= \frac{7(9\sqrt{3}-11\sqrt{2})}{9\sqrt{3}-11\sqrt{2}} = 7.$$

Ответ: 7.